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AN INDUCTION THEOREM FOR DISCOVERING SYNTACTIC TRANSLATIONS.(U)

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AN INDUCTION THEOREM FOR DISCOVERING SYNTACTIC TRANSLATIONS.

by

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ABSTRACT

Given an input-output sequence of syntactic translations of sentences generated by a deterministic finite state grammar G into Σ^*, a method is given for discovering the function which maps productions of G into Σ^* that gives rise to the observed translation.

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1. INTRODUCTION

Let $G = (V_N, V_T, P, S)$ be a right linear grammar [2]. Thus all productions in P are of the form

$$A \to aB \qquad \text{or} \qquad A \to a$$

where A and B are syntactic variables in V_N, and a is a terminal (or word) in V_T. We shall assume that G is *deterministic*, by which we mean that for every pair $(A, a) \in V_N \times V_T$ there is at most one production in P of the above form. We denote the set of sentences generated by G by L(G).

With G we shall associate what we shall call the *wiring diagram* $\mathcal{G}$ of G.

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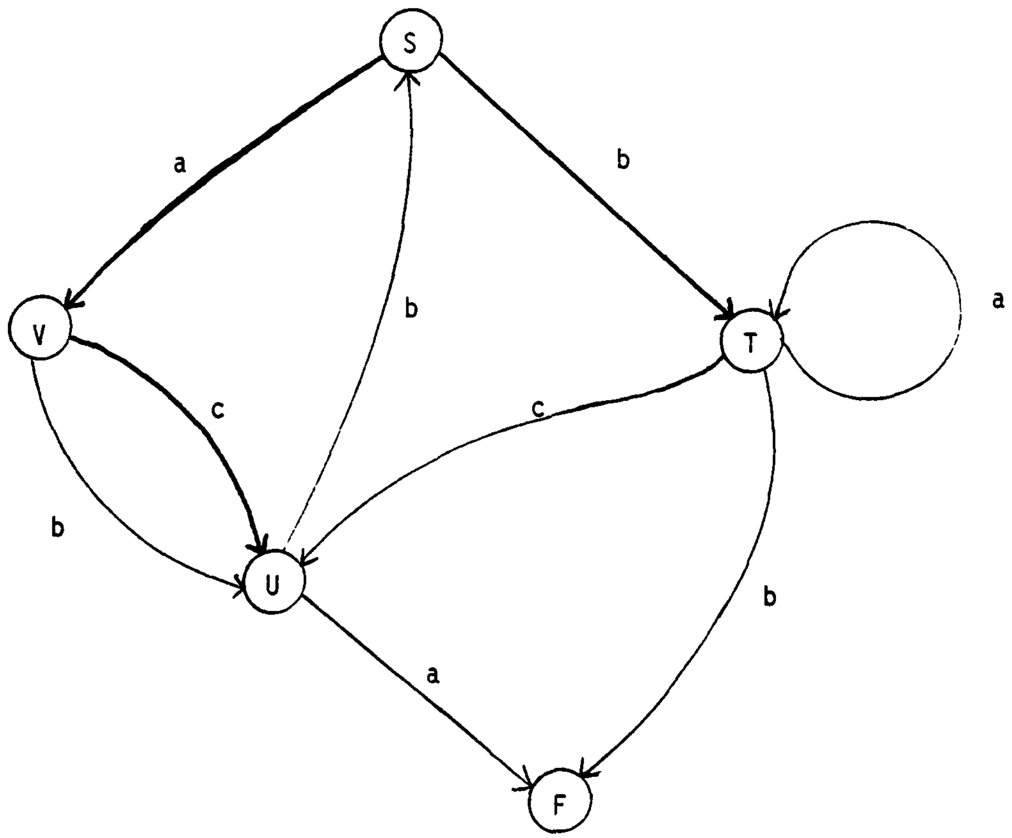
<u>Definition</u>. Let G be a right linear grammar. Then the wiring diagram $\mathcal{G}$ of G is a directed pseudograph [3] with labelled arcs. The node set $N(\mathcal{G})$ is $V_N \cup \{F\}$, where F is a symbol not in $V_N \cup V_T$. The arc set $A(\mathcal{G})$ is determined by the productions of G: if $A \to aB$ is an element of P then $A \xrightarrow{a} B$ is a labelled arc of $\mathcal{G}$; if $A \to a$ is an element of P then $A \xrightarrow{a} F$ is a labelled arc of $\mathcal{G}$.

For example, if $G = \left\{\{S, T, U, V\}, \{a, b, c\}, P, S\right\}$ where $P = \{S \to aV|bT,\ T \to aT|cU|b,\ U \to bS|a,\ V \to cU|bU\}$, then $\mathcal{G}$ is shown in Figure 1.



<u>Figure 1.</u>

There is obviously a natural correspondence between the elements of $L(G)$ and the set of walks from S to F in G; i.e., $L(G) = \{x_1 \ldots x_n | S \xrightarrow{x_1} X_1, X_1 \xrightarrow{x_2} X_2, \ldots, X_{n-1} \xrightarrow{x_n} F$ are labelled arcs of G, for some $X_1, \ldots, X_{n-1} \in V_N\}$. We shall assume throughout this paper that for each $A \in V_N$ in G there is a path from S to F that passes through A.

<u>Definition</u>. Given a deterministic right linear grammar G and a finite abstract set of symbols $\Phi = \{\phi_1, \ldots, \phi_s\}$, a <u>syntactic translation</u> is a map f from $A(G)$ to Φ^*.

If $A \xrightarrow{a} B$ is a labelled arc of G and if the image of this arc under f is ϕ where $\phi \in \Phi^*$, then graphically we write

$$A \xrightarrow{a|\phi} B$$

(Φ^* is the set of finite length sequences from Φ, including Λ, the empty string).

This definition is basically equivalent to the definition of a <u>generalized sequential machine</u> (gsm) [1], where f is called an <u>output function</u>.

By extending the definition of f in the natural way we have

$$f^{ex}: L(G) \to \Phi^* ;$$

i.e., if we have under f

$$S \xrightarrow{a_1|\phi^{(1)}} A_1, \ldots, A_{n-1} \xrightarrow{a_n|\phi^{(n)}} F$$

with $\phi^{(1)}, \ldots, \phi^{(n)} \in \Phi^*$, then the sentence

$$a_1 a_2 \ldots a_n \xrightarrow{f^{ex}} \phi^{(1)} \phi^{(2)} \ldots \phi^{(n)} .$$

In the syntactic translation as shown in Figure 2,

$$ba^2b \rightarrow \phi_5\phi_4\phi_5\phi_5\phi_4\phi_1$$

$$acbaba \rightarrow \phi_3\phi_1\phi_3\phi_1\phi_2\phi_3\phi_1\phi_3\phi_1\phi_1\phi_2 \text{ ,}$$

etc.

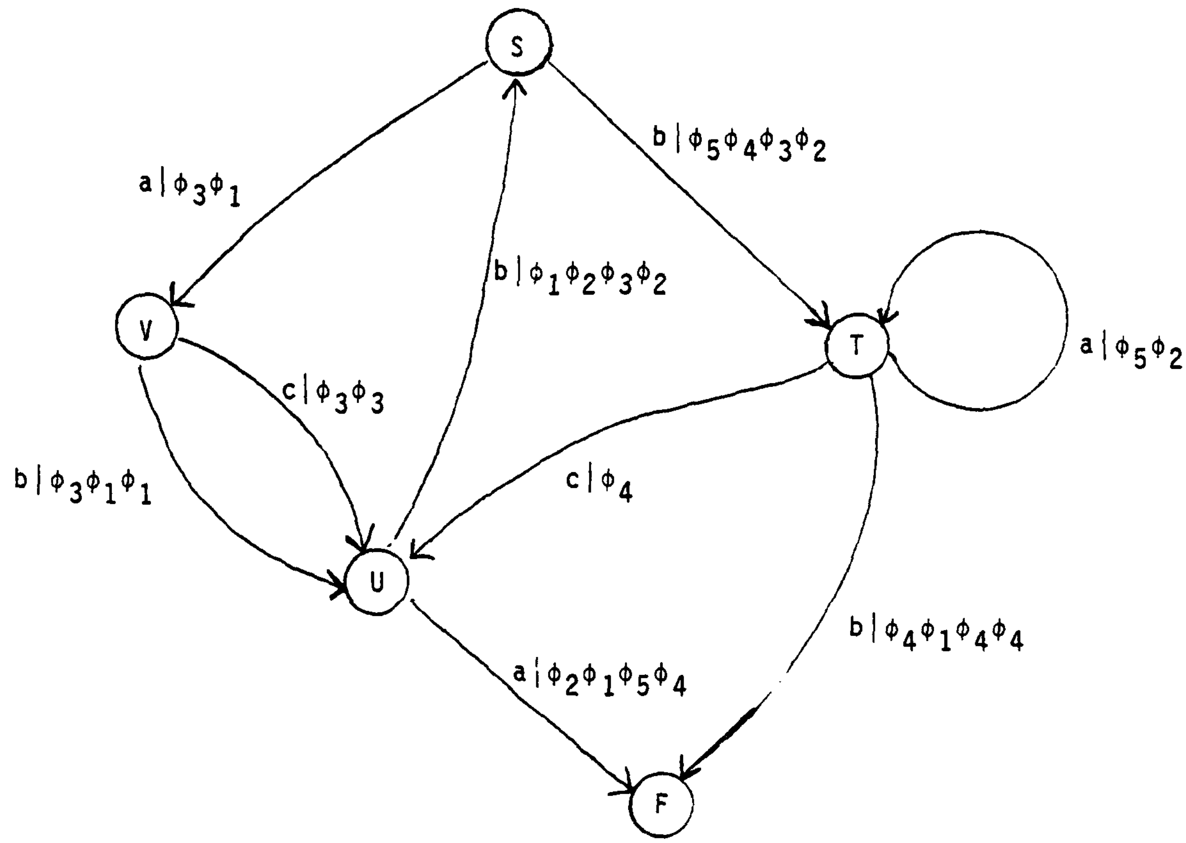


Figure 2.

Let $A(G, \Phi^*)$ be the set of syntactic translations of G, and let $A^{ex}(L(G), \Phi^*)$ be the extension of A to $(\Phi^*)^{L(G)}$. We shall refer to elements of $A^{ex}(L(G), \Phi^*)$ as <u>syntactic maps</u>.

2. TREE COMPOSITIONS

Definition. Let Σ be a finite alphabet, and $x \in \Sigma^*$. A k-composition of x is defined to be an ordered k-tuple $c \in (\Sigma^*)^k$, $c = (c_1,\ldots,c_k)$ having the property that $c_1c_2 \ldots c_k = x$. The set of k-compositions of x is denoted $C_k(x)$.

For example, if $\Sigma = \{a,b,c\}$, then $C_3(ab^2c)$ is the set $\{(\Lambda,\Lambda,ab^2c), (\Lambda,a,b^2,c), (\Lambda,ab,bc) \ldots\}$ where Λ denotes the empty word. In general, $|C_k(x)| = \binom{n+k-1}{k-1} = \binom{n+k-1}{n}$ if $|x| = n$.

The notion of composition is extended to trees.

Definition. Let Σ be a finite alphabet, T a rooted directed tree $T = (N(T), A(T))$. Thus T is a directed tree with a distinguished node $R \in N(T)$, and for each node $N \in N(T)$ there is a unique directed path from R to N. The leaves of T, denoted $L(T) \subset N(T) - R$ are the nodes of T with degree 1. Assume the elements of $L(T)$ are ordered $L_1,\ldots,L_\ell$ where $\ell = |L(T)|$. For a given element $x = (x_1,\ldots,x_\ell) \in (\Sigma^*)^\ell$ a T-composition of x is defined by a function

$$A(T) \xrightarrow{t^c} \Sigma^*$$

having the property that for each leaf L_j of T, and unique path $a_1,\ldots,a_k \in A(T)$ from R to L_j,

$$t^c(a_1)t^c(a_2) \ldots t^c(a_k) = x.$$

Thus a tree composition reduces to a k-composition when the tree is a rooted path consisting of k connected arcs. An example of a tree composition of (ab,ab,b,ba) is shown in Figure 3, for the complete binary tree with 7 nodes. Given T, along with an ordering for the leaves, and $x \in (\Sigma^*)^{L(T)}$ we denote the set of all tree compositions of x by $TC(T,x)$.

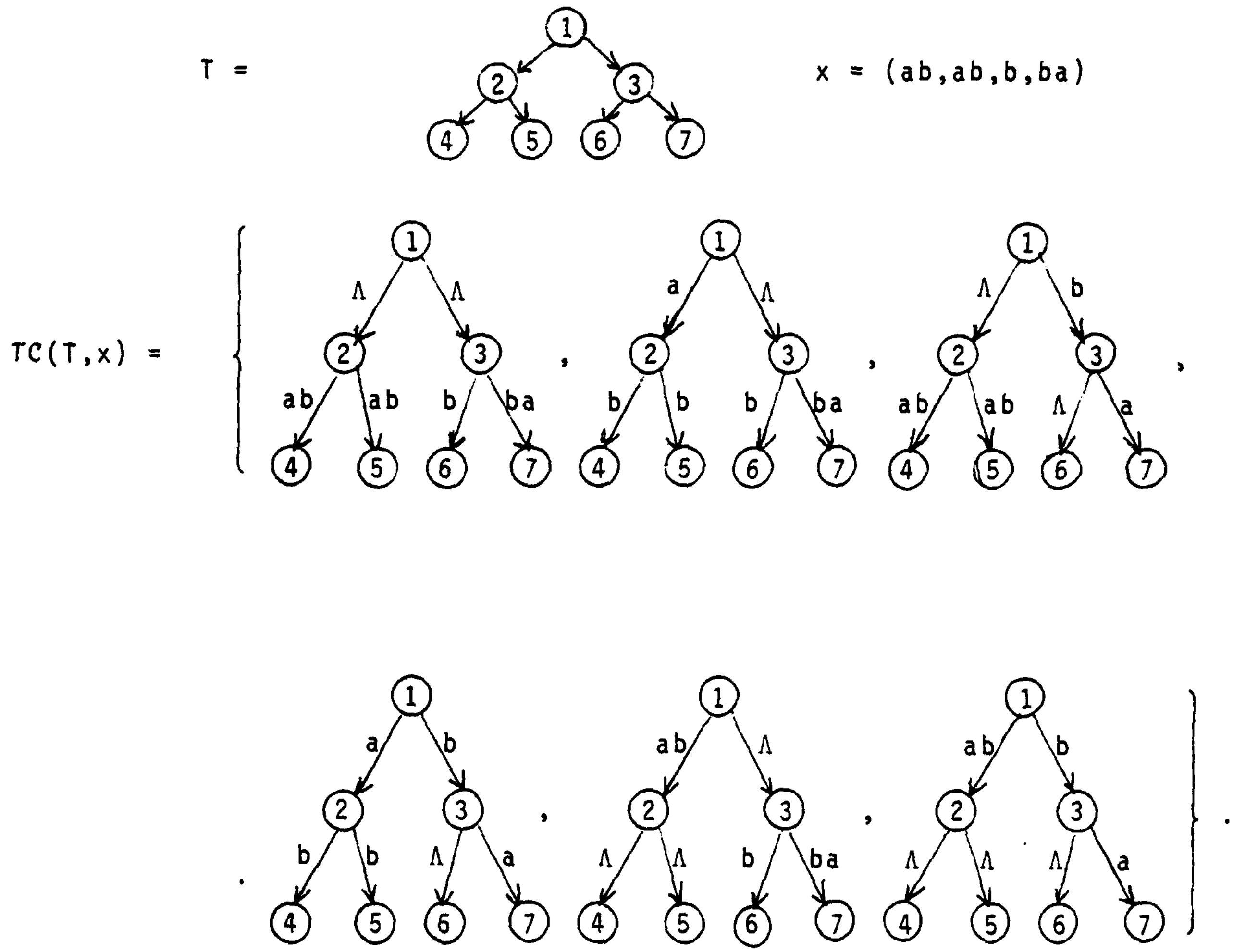


Figure 3.

An element of $TC(T,x)$ can be represented as a non-negative integer lattice point in a natural way:

If $a_1, \ldots, a_{|A(T)|}$ is some ordering of the arcs, then

$$t^c(a) \longrightarrow |t^c(a)| \qquad a \in A(T)$$

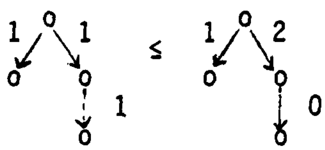
specifies a lattice point in $L = \mathbb{N}^{|A(T)|}$, $\mathbb{N}$ = non-negative integers.

We denote by $S[TC(T,x)]$ the set of lattice points defined above. A partial order $\leq_T$ is defined in L : for $s,t \in L$

$$s \leq_T t \quad \text{iff} \quad t \text{ is obtained from } s$$

by moving objects up the tree.

For example,



$$, \quad o \xrightarrow{1} o \xrightarrow{1} o \;\leq\; o \xrightarrow{2} o \xrightarrow{0} o \text{ , etc.}$$

We define, for $S \subset L$, max S = the elements of S having the property that for no $t \in S$: $s \leq t$, $t \neq s$.

3. THE INDUCTION PROBLEM

It is possible for two distinct syntactic translations to be extended to the same syntactic map. Thus we define an equivalence relation, ~ , on $S(G,\Phi^*)$ by defining $f_1 \sim f_2$ iff f_1 and f_2 are extended to the same element of $S^{ex}(L(G),\Phi^*)$.

The induction problem for syntactic translations is this: an observer O, who we assume knows the internal structure of the wiring diagram G except for the syntactic translation, can observe sentences from $L(G)$ along with their image in Φ^* under the unknown syntactic translation. Thus he can observe the syntactic map for a few sentences in $L(G)$. O wishes to discover an element $f \in S(G,\Phi^*)$ (up to equivalence) such that f^{ex} holds. We assume O can pick the sentences he wishes to observe. The theorem that follows shows, essentially, that O can pick a finite number of sentences from L(G) from which syntactic translation discovery is possible.

THEOREM: The syntactic translation (up to equivalence) can be discovered by observing a finite number of sentences W.

Remark: What the theorem says is that on observing a finite set W (to be constructed below), $\mathcal{O}$ is presented with a finite number of word equations:

$$(E)\qquad \begin{aligned} &a_{11}a_{12}\cdots a_{1i_1} = \phi(1)\\ &\vdots\\ &a_{k1}a_{k2}\cdots a_{ki_k} = \phi(k)\end{aligned}$$

where $|W| = k$, $a_{mn} \varepsilon A(G)$ (the arc set of G) and $\phi_{(j)}$ the observed image in Φ^* corresponding to the sentence determined by the walk $a_{j1} \ldots a_{ji_j}$ in G. A solution of E (that is, an assignment of values in Φ^* to the arcs $A(G)$ so that E is satisfied) will solve the induction problem.

Proof: The proof follows the construction of the implicit functions in [4].

We construct a spanning tree T in G, rooted at S and connecting all nodes in V_N. F is not connected to the spanning tree. For the example of Figure 1, a spanning tree T is indicated by darkened lines.

Label the arc set $A(G)$ in such a way that $A(T)$, the set of arcs in the spanning tree are $a_1,\ldots,a_t$.

From Φ and $A(T) = \{a_1,\ldots,a_t\}$ we create a new set of symbols. In general let X be a finite alphabet $\{x_1,\ldots,x_n\}$. Then define X^o *to be the group freely generated by the symbols of* X, with Λ the identity element. Form $(\Phi \cup A(T))^o$.

Begin at F and consider all arcs a entering F. Call this set $A(F)$, $A(F) \neq \phi$. Take an element a in $A(F)$. In what follows if a is the arc $A \xrightarrow{x} F$ then $\alpha(a) = A$, $\omega(a) = F$. Thus $\alpha(a) \in V_N$ and thus there is some walk $w = a_{i_1},\ldots,a_{i_j}, a$ from S to F with $a_{i_1},\ldots,a_{i_j} \in A(T)$. The sentence determined by the walk w, call it s, is mapped to $\phi(s)$, which $\mathcal{O}$ observes and writes

$$a = a_{i_j}^{-1} \ldots a_{i_1}^{-1}\phi \in (\Phi \cup A(T)^o) .$$

This is done for each element of $A(F)$.

$\mathcal{O}$ now considers the arcs of $A(G) - (A(T) \cup A(F))$. Let $A^{(j)} =$ the set of arcs a of G not in $A(T)$ such that the number of arcs in the shortest path (a walk with no repeated nodes) from $\omega(a)$ to F is j (i.e., $A^{(0)} = A(F)$). Suppose $\mathcal{O}$ has computed the equations for the arcs in $A^{(0)},\ldots,A^{(j-1)}$. Let $a \in A^{(j)}$ and let $a,b_1,\ldots,b_j$ be a shortest path from $\omega(a)$ to F. Now $\alpha(a) \in V_N$ hence

$$a_{i_1} \ldots a_{i_j}\, \underline{a}\, b_1 \ldots b_j,$$

a walk from S to F, $a_{i_1},\ldots,a_{i_j} \in A(T)$. If this corresponds to sentence s then $\mathcal{O}$ observes $\phi(s)$, so that $a = a_{i_j}^{-1} \ldots a_{i_1}^{-1}\, \phi\, b_j^{-1} \ldots b_1^{-1} \in (\Phi \cup A(T))^o$ by using the equations for $b_1,\ldots,b_j$ from previous computations. This process terminates with a list of equations

$$(I)\begin{cases} a_{k+1} = g_1 \\ \vdots \\ a_q \quad = g_{q-k} \end{cases}$$

where $g_1, \ldots, g_{q-k}$ are elements of $(\Phi \cup A(T))^0$.

For example, from Figure 3 if we define the arcs

$$\left.\begin{array}{ll} a_1 & S \xrightarrow{a} V \\ a_2 & S \xrightarrow{b} T \\ a_3 & V \xrightarrow{c} U \end{array}\right\} \text{ spanning tree} \;.$$

$$\begin{array}{ll} a_4 & U \xrightarrow{a} F \\ a_5 & T \xrightarrow{b} F \\ a_6 & T \xrightarrow{a} T \\ a_7 & T \xrightarrow{c} U \\ a_8 & U \xrightarrow{b} S \\ a_9 & V \xrightarrow{b} U \end{array}$$

Then

$$\begin{aligned}
a_1 a_3 \underline{a_4} &= \phi_3 \phi_1 \phi_3^2 \phi_2 \phi_1 \phi_5 \phi_4 \\
a_2 \underline{a_5} &= \phi_5 \phi_4 \phi_3 \phi_2 \phi_4 \phi_1 \phi_4^2 \\
a_2 \underline{a_6} a_5 &= \phi_5 \phi_4 \phi_3 \phi_2 \phi_5 \phi_2 \phi_4 \phi_1 \phi_4^2 \\
a_2 \underline{a_7} a_4 &= \phi_5 \phi_4 \phi_3 \phi_2 \phi_4 \phi_2 \phi_1 \phi_5 \phi_4 \\
a_1 \underline{a_9} a_4 &= \phi_3 \phi_1 \phi_3 \phi_1^2 \phi_2 \phi_1 \phi_5 \phi_4 \\
a_1 a_3 \underline{a_8} a_2 a_5 &= \phi_3 \phi_1 \phi_3^2 \phi_1 \phi_2 \phi_3 \phi_2 \phi_5 \phi_4 \phi_3 \phi_2 \phi_4 \phi_1 \phi_4^2 \;.
\end{aligned}$$

These equations can be solved in the group $(\Phi \cup A(T))^0$ by the method indicated.

It follows from [4] that, given (I), the syntactic map is the same for all assigments of $a_1, \ldots, a_k$ to elements of Φ^0, and hence Φ^*. What this means is that, given the finite equations (I), an assignment of values in Φ^* to the arcs of the spanning tree $a_1,\ldots,a_k$ so that $a_{k+1},\ldots,a_q$ as defined by (I) are in Φ^* will solve the induction problem. [].

A sequence $a_1,\ldots,a_k \in \Phi^*$ such that $a_{k+1},\ldots,a_q$ are in Φ^* is called a <u>feasible point</u>.

4. THE INDUCTION SOLUTION

The structure of equations (I) will help in solving the word equations. Instead of the equation $a_{k+r} = g_r$ in (I) let us consider its associated equation $r = 1,\ldots,g-k$

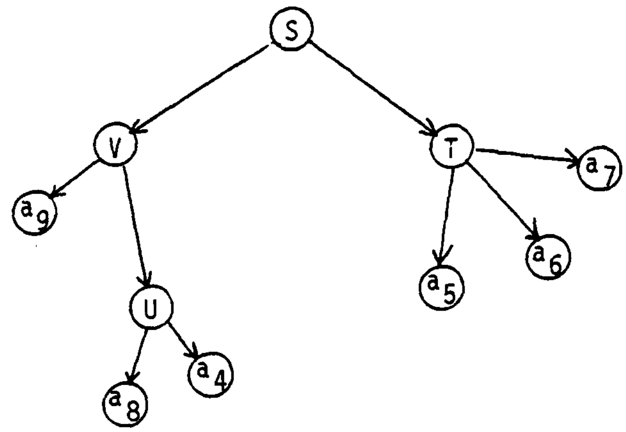
$$\phi_{(r)} = a_{i_1}\ldots a_{i_j}\ \underline{a_{k+r}}\ b_1 \ldots b_j$$

as determined in the proof of Theorem 1. Thus $a_{i_1} \ldots a_{i_j}$ denotes a descent down the spanning tree T, a_{k+r} the unknown in (I), $b_1 \ldots b_j$ a shortest path from $w(a_{k+r})$ to F.

From T we shall construct a new tree T' by adding leaves to T as follows. The new leaves will be labelled $a_{j+1},\ldots,a_q$ and will be directed respectively to the nodes

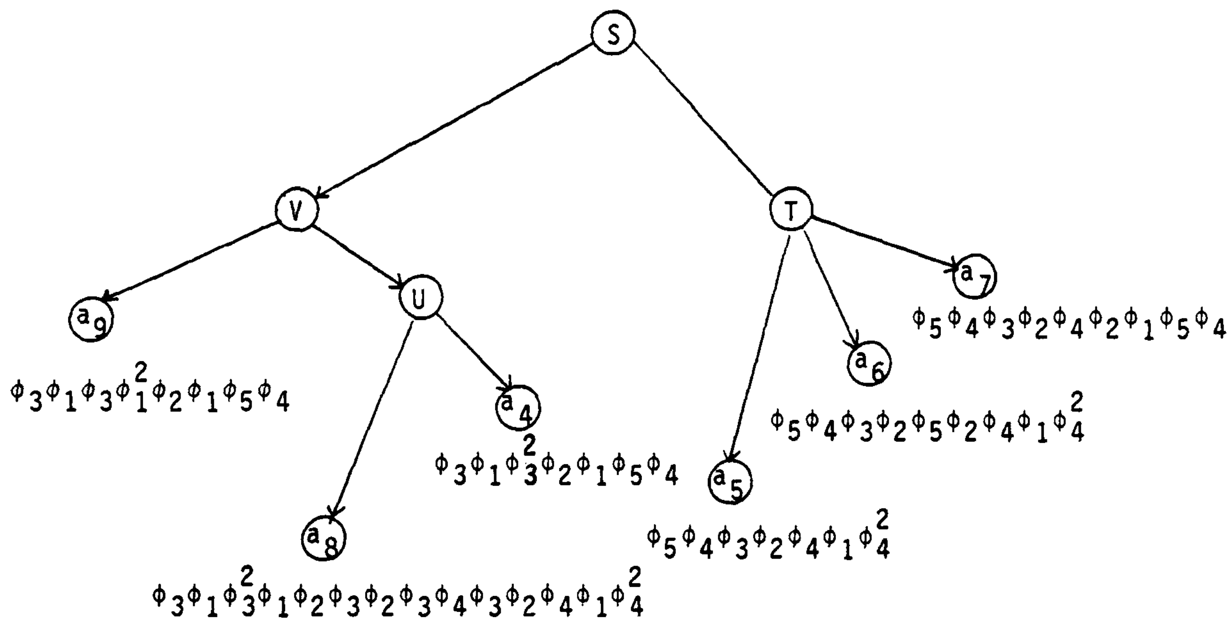
$$\alpha(a_{j+1}),\ldots,\alpha(a_q) .$$

Thus the spanning tree T of Figure 1 becomes T' in Figure 4. If we consider $TC(T',x)$ where $x \in (\Phi^*)^{q-k}$ $x = (\phi_{(1)},\ldots,\phi_{(q-k)})$ is the vector of observed sentences from Φ^*, then obviously the set of feasible points $a_1,\ldots,a_k$ are in $TC(T',x)\Big|_{a_1,\ldots,a_k}$; that is, $TC(T',x)$ restricted to the arcs $a_1,\ldots,a_k$. In some examples it turns out that a feasible point can be discovered by computing $\max[TC(T',x)]$, but this is not always the case. Consider Figures 5 and 6.

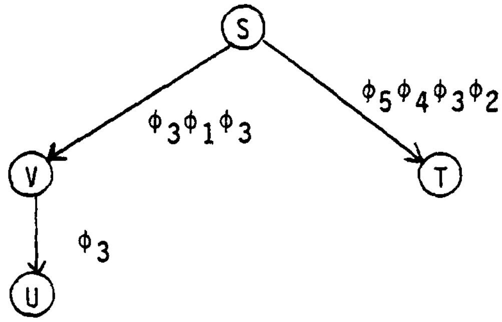


T'

<u>Figure 4</u>.



<u>Figure 5</u>.



$$\max TC(T',x)\Big|_{a_1,a_2,a_3}$$

Figure 6.

Figure 6 gives $\max TC(T',x)\Big|_{a_1,a_2,a_3}$, a feasible point (which is easily verified).

Figure 7 gives an example of a case where $\max TC(T',x)\Big|_{a_i \varepsilon T}$ is not a feasible point.

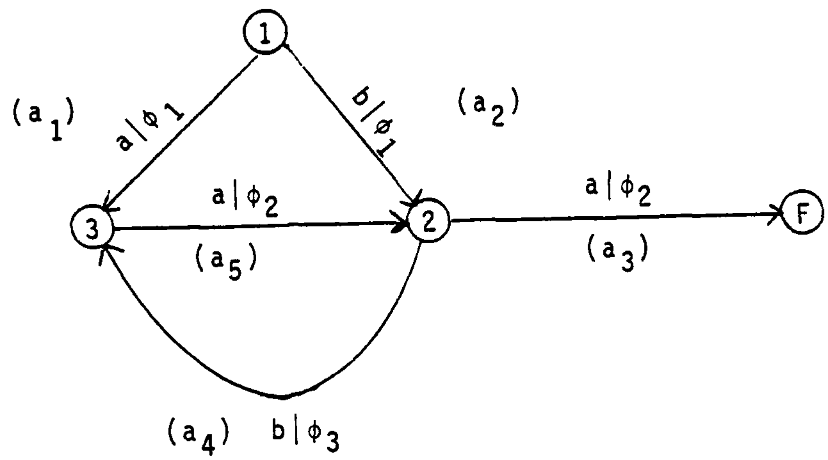
An obvious necessary condition, in addition to the feasible points being in $TC(T',x)\Big|_{a_i \varepsilon T}$, is

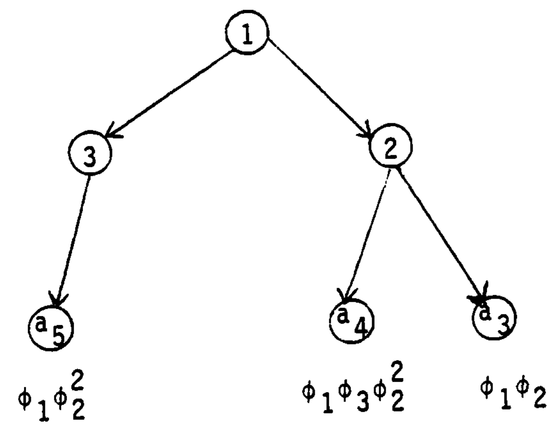
$$|\phi_{(r)}| = \left|a_{i_1}\right| + \ldots + \left|a_{i_j}\right| + |a_{\underline{k+r}}| + |b_1| + \ldots + |b_j| .$$

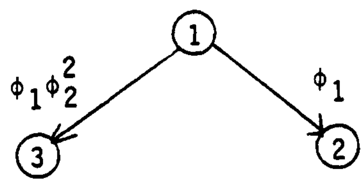
Note for the example in Figure 7, if we let $|a_i| = x_i$ then

$$\begin{aligned} x_2 + \underline{x_3} \quad\quad\quad\; &= 2 \\ x_1 + \underline{x_5} + x_3 \quad\; &= 3 \\ x_2 + \underline{x_4} + x_5 + x_3 &= 4 . \end{aligned}$$

If $x_1 = 3$ and $x_2 = 1$, as we have in the $\max TC(T',x)\Big|_{a \varepsilon T}$ solution, then there is no (x_3, x_4, x_5) *non-negative solution*.







$\max TC(T',x)\Big|_{a_i \in T}$

Figure 7.

As before, we denote the word equation for the variable a_{k+r} by $(a_{k+r} \in A^{(j)})$

$$a_{i_1} \dots a_{i_\ell} \underline{a_{k+r}} b_1 \dots b_j = \phi_{(r)} .$$

Let us now assume that $b_1 \dots b_j$ (a shortest path from $w(a_{k+r})$ to F) is chosen so that it is a suffix of a previously defined walk.

<u>THEOREM</u>: A sufficient condition for an assignment of arcs $a \in T$ to values in Φ^* to be feasible is that it satisfies

$$\max TC(T',\phi)$$

subject to

$$(*) \quad |w_{(j)}| = \phi_{(j)}$$

where $w_{(j)}$ is the walk from S to F corresponding to the variable a_{k+j} .

<u>Proof</u>: Let $\hat{\phi}(a)$, $a \in A(G)$, be the "true" unknown syntactic translation, so for $r = 1,\dots, g-k$

$$\hat{\phi}(a_{i_1}) \dots \hat{\phi}(a_{i_\ell})\hat{\phi}(a_{k+r})\hat{\phi}(b_1^{(r)}) \dots \hat{\phi}(b_j^{(r)}) .$$

Let $\phi(a)\Big|_{a \in T}$ be the assignment determined by the criteria stated in the theorem.

We claim that for each $s = 1,\dots, \ell$

$$\phi(a_{i_s}) \dots \phi(a_{i_\ell}) \; \phi(a_{k+r}) \dots \phi(b_j)$$

is a suffix of

$$\hat{\phi}(a_{i_s}) \dots \hat{\phi}(a_{i_\ell}) \; \hat{\phi}(a_{k+r}) \dots \hat{\phi}(b_j) .$$

If this were not true, then we would have, for some s ,

$$\phi(a_{i_1}) \dots \phi(a_{i_{s-1}})$$

being a proper prefix of

$$\hat{\phi}(a_{i_1}) \ldots \hat{\phi}(a_{i_{s-1}})$$

and this contradicts maximality.

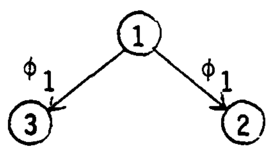
Consequently, $\phi(b_1) \ldots \phi(b_j)$ is a suffix of $\phi_{(r)}$ (by induction, $b_1 \ldots b_j$ is of the form $a'_{i_s} \ldots a'_{i_\ell}\ a'_{k+r}\ b'_1 \ldots b'_j$ for a previously computed walk) $\phi(a_{i_1}) \ldots \phi(a_{i_\ell})$ is a prefix of $\phi_{(r)}$, so by (*) we have a solution in Φ^* of $\phi(a_{k+r})$. []

The example of Figure 7 shows that

$$\begin{cases} x_2 + x_3 = 2 \\ x_1 + x_5 + x_3 = 3 \\ x_2 + x_4 + x_5 + x_3 = 4 \end{cases}$$

$$\Rightarrow (x_1,x_2) \in \{(0,0),(0,1),(1,0),(1,1),(1,2)\} .$$

$(x_1,x_2) = (1,1)$ corresponds to



$$\max TC(T',x)\Big|_{a \in T}$$

subject to (*)

which is indeed feasible.

It is evident that we may replace $TC(T',\phi)$ with a set of inequalities, i.e., for the example in Figure 7 we must have

$$x_1 \le 3$$
$$x_2 \le 1 \ ,$$

for the example in Figure 5

$$x_1 \le 3$$
$$x_2 \le 4$$
$$x_1 + x_3 \le 5 \ .$$

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Given an input-output sequence of syntactic translations of sentences generated by a deterministic finite state grammar G into Σ^*, a method is given for discovering the function which maps productions of G into Σ^* that gives rise to the observed translation.